

# Transversity $K$ Factors for Drell–Yan Processes

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**Abstract.** The question of the  $K$  factor in transversely polarised Drell–Yan (DY) processes is examined. The transverse-spin case is peculiar for the absence of a reference point in deeply inelastic scattering (DIS). Therefore, in order to study more fully the possible effects of higher-order corrections on DY asymmetries, a DIS definition for transversity is devised using a hypothetical scalar (Higgs-like) vertex. The results show that some care may be required in interpreting experimentally extracted partonic transversity, in particular when comparing with model calculations or predictions.

## 1 Introduction

The theoretical framework for describing transversity (at the basic level of partonic processes, QCD evolution, radiative effects *etc.*) is now solid [1] and a number of experiments aimed at its measurement are on-line or under development: HERMES [2], COMPASS [3] and the RHIC spin programme [4]; there are also proposals for Drell–Yan (DY) measurements with polarised antiprotons in the High Energy Storage Ring at GSI [5, 6] (related preliminary theoretical studies have been made regarding access to transversity in  $J/\psi$  production [7, 8]).

Transversity is the last remaining piece in the partonic jig-saw puzzle composing the hadronic picture. However, the standard procedure of adopting deeply inelastic scattering (DIS) as the process to *define* parton densities at the next-to-leading order (NLO) cannot be extended to transversity in a simple manner since it does not contribute to DIS. Furthermore, transverse-spin effects are notoriously surprising; *e.g.*, see the large and (historically) unexpected single-spin asymmetries [9, 10]. Such considerations render imperative the complete understanding of NLO corrections in DY before attempts are made to extract the partonic transversity distributions. See, *e.g.*, Ref. [11] for a detailed discussion of transversity and also single-spin asymmetries.

One might instead consider double-spin asymmetries  $A_{TT}$  for other processes, such as:  $p^\uparrow p^\uparrow \rightarrow \text{jet} + X, \gamma + X$  *etc.* Unfortunately, however, predictions for  $A_{TT}$  always turn out to be very small [12, 13], so that measuring transversity directly appears feasible only in doubly polarised  $p\bar{p}$  interactions.

Since all QCD and electroweak vertices conserve quark chirality, transversity actually decouples from DIS. Chiral-

ity flip is not a problem though if the quark lines connect to different hadrons as in, *e.g.*, the DY process. Unfortunately, there is a *caveat* to accessing transversity in DY: Hikasa’s theorem [14], which states that, owing to chiral symmetry, transversity effects vanish upon integrating over the lepton-pair azimuth. No simple proof of the theorem exists (it has to do with the  $\gamma$ -matrix algebra). Let us now make a few observations based on these properties of transversity:

1. the only “gold-plated” process in which transversity may be measured directly (*i.e.*, without the need of more-or-less exotic fragmentation functions) is DY;
2. Hikasa’s theorem implies the use of a slightly less than fully inclusive process, in as much as one angle must be left unintegrated;
3. in the case of transversity asymmetries, helicity conservation may not necessarily provide the usual safeguard against large  $K$  factors.

The above have non-trivial implications with respect to the measurement of transversity in DY and interpretation of the results.

1. In the absence of a DIS reference point, there is no immediate way of fully evaluating the possible importance of higher-order QCD corrections. The  $K$  factors are known to be large at the level of cross-sections in both the unpolarised [15] and helicity-dependent [16] cases. However, in the helicity case the large corrections cancel in the asymmetry [16]. To a large extent this cancellation can be traced to the conservation of helicity along fermion lines in gauge theories—the  $O(\alpha_s)$  Wilson coefficient for the DY process is identical for the helicity-dependent and

-independent pieces, as too are the leading order (LO) anomalous dimensions. Note that in the case of heavy-flavour production the corrections to the helicity asymmetry are large, precisely because mass terms introduce helicity flip, destroying the usual protection.

**2 + 3.** The coefficient function for transversely polarised DY differs significantly from the other two cases [17, 18]. Moreover, the LO anomalous dimensions differ—there is no corresponding conserved quantity or sum-rule.

Given the marked differences from the other two cases, it may be useful to examine the question of DY  $K$  factors for transversity. In order to do this, it is clearly necessary to find some suitable DIS-like process as a reference point. The principal requirement is a spin-flip mechanism. There are two obvious possibilities *a priori*: either a quark mass term or a scalar vertex. Now, of course, DIS with transversely polarised leptons and nucleons should be considered and therefore the twist-three structure function  $g_2$  (the relevant operator is indeed proportional to the quark mass [21–23]), at the level of direct contribution to polarised DIS it actually cancels against other higher-twist contributions owing to the equations of motion, see for example Ref. [24]. Although the calculation is rather delicate, the possibility of defining a coefficient function for transversity via its rôle in the evolution of  $g_2$  has been examined [25], with similar results to those presented here.

A simpler and more direct approach is to identify a DIS-like process in which a scalar particle plays a rôle. Since the Higgs boson does indeed interact with quarks (as with leptons too), the obvious solution to the problem is a *gedanken* process in which the exchange is no longer via the electroweak gauge fields but via the Higgs particle. To be precise, in order to obtain the required single spin-flip, Higgs–Vector interference diagrams actually need to be considered. Of course, there is no intended suggestion here that such a process should really be measured, but merely that it forms a suitable basis for a theoretical cross-check. We should remark that such a process has effectively already been exploited for the calculation of  $h_1(x)$  itself [26], on the basis of a suggestion by Jaffe. In any case, various tests will be performed to ensure that the results do not depend on the specific nature of the vertex introduced.

Before moving on to the calculation, let us spend a few more words on the physical significance of the  $K$ -factor. While at a theoretical level the meaning of higher-order corrections to any given process is clear, at a phenomenological level in the parton-model there is an inherent ambiguity owing to the necessary input of the parton densities themselves. Indeed, the  $K$ -factor was used historically to represent the discrepancy between experimental results for the DY cross-section and the LO theoretical predictions based on parton densities extracted from DIS and thus *by definition* (as in Refs. [15, 16]) the phenomenological  $K$ -factor is the translation factor from DIS to DY.

Therefore, since all model calculations or estimates of partonic transversity densities rely to some extent on DIS for overall normalisation or determination of model parameters, self-consistency would require a procedure of the type to be described here. Since, furthermore, the overall  $K$ -factor so-defined receives large contributions from *both* DIS *and* DY, this is a non-trivial observation. The peculiar structure of transversity leaves room for very different corrections as compared to the spin-averaged or helicity-dependent cases, for both DIS and DY independently.

Thus, in the following section the calculations are described, the Higgs–Vector interference mechanism is examined in detail and NLO calculation of the related Wilson coefficients is performed. The known results for the DY process are discussed and finally the relevant  $K$  factors are extracted. In the closing section some conclusions are drawn and comments relevant to future measurements of transversity via DY scattering are made.<sup>1</sup>

## 2 The Calculation

### 2.1 Drell–Yan cross-section and asymmetries

It is now standard to define the helicity- and transversity-weighted cross-sections by

$$\frac{d\Delta\sigma}{dQ^2} \equiv \frac{1}{2} \left[ \frac{d\sigma^{++}}{dQ^2} - \frac{d\sigma^{+-}}{dQ^2} \right] \quad (1a)$$

and

$$\frac{d\Delta_T\sigma}{dQ^2} \equiv \frac{1}{2} \left[ \frac{d\sigma^{\uparrow\uparrow}}{dQ^2} - \frac{d\sigma^{\uparrow\downarrow}}{dQ^2} \right], \quad (1b)$$

where the prefixes  $\Delta$  and  $\Delta_T$  indicate longitudinal-spin (or helicity) and transverse-spin (or transversity) dependence respectively,  $\pm$  refer to initial-state proton helicities and  $\uparrow, \downarrow$  to transverse polarisations. The double-spin asymmetries are then

$$A_{LL} \equiv \frac{d\Delta\sigma/dQ^2}{d\sigma/dQ^2} \quad (2a)$$

and

$$A_{TT} \equiv \frac{d\Delta_T\sigma/dQ^2}{d\sigma/dQ^2}. \quad (2b)$$

The large NLO corrections afflict both the numerators and denominators. The question is to what extent they are correlated, *i.e.*, to what extent they are the same and thus cancel in the ratio.

Turning then to the calculation of the  $K$  factor, the procedure will be essentially identical to that followed in earlier work [15, 16] and thus we shall not dwell on the general technicalities, save for those points that are significantly different in the case of transverse polarisation. The first peculiar aspect to be exploited is that, owing to

<sup>1</sup> Owing to correction of an error in the code used for numerical estimates, the results shown here are a little less dramatic than those presented by the author in past conferences.

the charge-conjugation properties of the relevant operator, the evolution of transversity is of the flavour *non-singlet* (NS) type. In the NS case the effect of higher-order corrections may be represented in the following schematic way:

$$F(x, t) = \int_x^1 \frac{dy}{y} \sum_f Q_f^2 \left[ \delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s(Q^2)}{2\pi} t P\left(\frac{x}{y}\right) + \frac{\alpha_s(Q^2)}{2\pi} C\left(\frac{x}{y}\right) \right] q_f(y, t), \quad (3)$$

where  $t \equiv \ln(Q^2/\mu^2)$ , with  $Q^2$  the virtuality of the photon,  $q_f(y, t)$  and  $Q_f$  are respectively the parton density and charge of quark flavour  $f$ ,  $P$  is the universal quark–quark splitting function and  $C$  the process-dependent Wilson coefficient. The quantity  $F(x, t)$  on the left-hand side then represents a generic (flavour NS) structure function and the three terms inside the square brackets on the right-hand side represent: 1. the LO point-like contribution; 2. the leading-logarithmic correction; and 3. the NLO correction. It is this last that is of interest here.

To NLO the DY cross-section for  $p\bar{p}$  scattering is expressed in term of parton densities as follows:

$$Q^2 \frac{d\sigma^{\text{DY}}}{dQ^2} = \frac{4\pi\alpha}{9s} \int_0^1 dx_1 dx_2 dz \delta(x_1 x_2 z - \tau) \times \sum_f Q_f^2 \left[ q_f(x_1, Q^2) \bar{q}_f(x_2, Q^2) + (1 \leftrightarrow 2) \right] \times \left[ \delta(1 - z) + \frac{\alpha_s(Q^2)}{2\pi} C^{\text{DY}}(z) \right], \quad (4)$$

where  $\tau = Q^2/s$ ,  $Q^2$  is the invariant mass squared of the lepton pair and  $s$  is the total hadron centre-of-mass energy squared. In Eq. (4)  $x_{1,2}$  are the momentum fractions carried by the (anti)quarks inside hadrons 1 and 2 respectively. It is then the difference between the DIS corrections, with which the NLO parton distributions are defined via Eq. (3), and NLO DY corrections that constitutes the phenomenological  $K$  factor.

The leading-logarithmic splitting functions  $P$  are well known [27–32] and may be expressed in the following compact form:

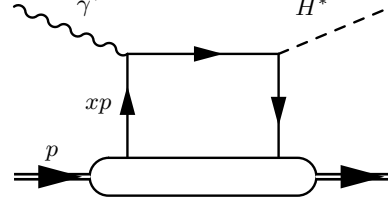
$$\Delta P(z) = P(z) = C_F \left[ \frac{1+z^2}{1-z} \right]_+ \quad (5a)$$

and

$$\Delta_T P(z) = P(z) - C_F(1 - z), \quad (5b)$$

The definition of the so-called “plus” regularisation is recalled in Appendix A. Already then it is evident that although fermion-helicity conservation guarantees identical evolution for NS spin-averaged and helicity-weighted quark densities, the same does not hold for the transversity case.

The problem now is to calculate the coefficient  $C(z)$  of the third term in Eq. (3). This must be done for both the DIS and DY processes. As is well-known, a large part of the DY  $K$  factor can be attributed to the change from



**Fig. 1.** The DIS “handbag” diagram for a photon–Higgs interference process.

a space-like  $Q^2$  in DIS to time-like in DY. However, this is not the only origin of large corrections and one should be concerned that the transversity case, with the extra requirement on the final-state phase space in DY, might introduce important differences.

## 2.2 The Drell–Yan Process

Since the results for the DY process are known [17, 18], it is perhaps better to begin with this coefficient. The virtual photon decays into a final lepton pair, of which then the azimuthal angle must be left unintegrated in the case of transversity. In the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme the results for the unpolarised [15], helicity [16] and transversity [18] DY coefficient functions are as follows:<sup>2</sup>

$$C^{\text{DY}}(z) = C_L^{\text{DY}}(z) = C_F \left\{ 4(1+z^2) \left[ \frac{\ln(1-z)}{1-z} \right]_+ - \frac{2(1+z^2) \ln z}{(1-z)} + \left[ \frac{2}{3} \pi^2 - 8 \right] \delta(1-z) \right\}, \quad (6a)$$

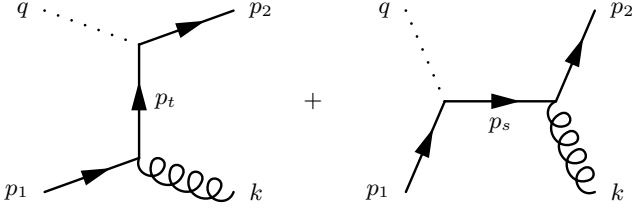
$$C_T^{\text{DY}}(z) = C^{\text{DY}}(z) + C_F \left\{ -4(1-z) \ln(1-z) + 2(1-z) \ln z - \frac{6z \ln^2 z}{(1-z)} + 4(1-z) \right\}. \quad (6b)$$

It is important to note that although the large  $\pi^2$  terms and indeed the coefficient of the  $\delta$ -function (which indeed constitute the bulk of the  $K$  factor) appear invariant here, in the transversity case there is a new term,  $-\frac{6z \ln^2 z}{(1-z)}$ , not found in the others.

## 2.3 Deeply Inelastic Scattering

In order to accommodate spin-flip in the standard DIS “handbag” diagram, one of the vertices should involve a Higgs-like scalar, see Fig. 1. The contribution of this di-

<sup>2</sup> Here and in what follows only the unpolarised result will be presented in full while the helicity and transversity cases will be shown as differences with respect to the unpolarised case.



**Fig. 2.** The two diagram types contributing to the NLO DIS hard partonic  $q\gamma^*(H^*) \rightarrow gq$  scattering subprocess, the dotted line represents either a virtual photon or Higgs.

agram can be expressed in terms of a structure that will be called  $h_1$  here for brevity. Projecting with  $\gamma_5 \not{p}_T$  then leads to

$$W^\mu = h_1(x, Q^2) \frac{i\epsilon^{qpsT\mu}}{p \cdot q}. \quad (7)$$

### 2.3.1 Real-gluon contributions

The NLO Wilson coefficient may now be calculated from the diagrams in Fig. 2. The process to be calculated is, of course, still photon–Higgs interference. The use of dimensional regularisation poses the problem of dealing with  $\gamma_5$ , which naturally arises in the case of polarised DIS (for both helicity and transversity) owing to the projector  $\gamma_5 \not{p}_T$ . Note that for the  $q\bar{q} \rightarrow \gamma^*g$  DY subprocess this is not a problem since both the quark and antiquark bring one power of  $\gamma_5$ , which then cancels before calculating any traces. The technique adopted here is that of defining a fully anti-commuting but non-cyclic  $\gamma_5$ , see for example Ref. [33]; see also Ref. [34] for a detailed discussion of this technique. Consistency then requires that all traces be evaluated from the same reference point, which in the usual DIS case is unambiguously one of the photon vertices. Here the scalar vertex could also be chosen—an explicit check shows that there is no ambiguity in the results.

In the  $\overline{\text{MS}}$  scheme (see Appendix B for a working definition), adopting the above-mentioned  $\gamma_5$  scheme and suppressing (for clarity) a common factor

$$C_F \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

on the right-hand side of all equations, the results for the real contributions of the diagrams in Fig. 2 are:

$$\begin{aligned} \tilde{C}^{\text{DIS-R}}(z) &= \frac{2}{\epsilon^2} \delta(1-z) - \frac{1}{\epsilon} \left[ \frac{1+z^2}{(1-z)_+} - \frac{3}{2} \delta(1-z) \right] \\ &+ (1+z^2) \left[ \frac{\ln(1-z)}{1-z} \right]_+ - \frac{3}{2} \frac{1}{(1-z)_+} + 3 + 2z \\ &- (1+z^2) \frac{\ln z}{(1-z)} + \frac{7}{2} \delta(1-z), \end{aligned} \quad (8a)$$

$$\tilde{C}_L^{\text{DIS-R}}(z) = \tilde{C}^{\text{DIS-R}}(z) - 1 - z, \quad (8b)$$

$$\begin{aligned} \tilde{C}_T^{\text{DIS-R}}(z) &= \tilde{C}^{\text{DIS-R}}(z) + \frac{1}{\epsilon}(1-z) \\ &- (1-z) \ln \left( \frac{1-z}{z} \right) - \frac{3}{2} - 2z, \end{aligned} \quad (8c)$$

where the  $\tilde{C}^{\text{DIS-R}}(z)$  are defined to be the combined quantities

$$\tilde{C}^{\text{DIS}}(z) = t P(z) + C^{\text{DIS}}(z), \quad (9)$$

with  $P(z)$  and  $C(z)$  being replaced respectively by  $\Delta P(z)$  and  $C_L(z)$  etc., where necessary. In the first equation, for  $\tilde{C}(z)$ , an additional contribution  $3z$  due to  $F_L$  has been included to give the correction corresponding to the use of  $F_2$  to define  $q(x)$  [15]. Moreover, in Eq. (8c) the remaining  $\epsilon$  is due to the difference in splitting functions and disappears in the final expression for the full coefficient.

To extract the desired coefficients  $C^{\text{DIS-R}}(z)$ , the virtual corrections must now, of course, also be added. First however, note that the results for the unpolarised and helicity cases agree with previous calculations, [15] and [16] respectively. Note also that the results for the various cases are (not surprisingly) similar: while the coefficient for  $h_1$  is a little different (owing to the finite residues of the ultraviolet divergences, which lead to different splitting functions), the infrared (IR) double poles in  $\epsilon$  are identical (and in any case cancel with the virtual contributions) and the single poles themselves are, of course, to be absorbed into the scale-dependent parton densities. In particular, there is no trace of the  $\frac{6z \ln^2 z}{(1-z)}$  term found in the DY coefficient.

### 2.3.2 Virtual-gluon contributions

The virtual contributions can be partially gleaned from the literature; however, the scalar-vertex correction remains to be evaluated and this requires a some care. The real and virtual contributions are separately gauge invariant and a natural choice (as in Refs. [15, 16] and other cited work) is the Landau gauge, where it is only the vertex correction that need be calculated. Schematically,

$$\Gamma_V^\mu(q^2) = \gamma^\mu \left[ 1 + \frac{\alpha_s}{2\pi} \delta_V \right] \quad (10a)$$

and

$$\Gamma_S(q^2) = \mathbb{1} \left[ 1 + \frac{\alpha_s}{2\pi} \delta_S \right], \quad (10b)$$

where  $\mathbb{1}$  represents the “bare” scalar vertex. In  $\overline{\text{MS}}$  the following results are then obtained:

$$\delta_V = C_F \left[ \frac{\mu^2}{-q^2} \right]^\epsilon \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 - \frac{2}{3}\pi^2 \right] \quad (11a)$$

and

$$\delta_S = C_F \left[ \frac{\mu^2}{-q^2} \right]^\epsilon \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ -\frac{2}{\epsilon^2} - 2 - \frac{2}{3}\pi^2 \right]. \quad (11b)$$

Noting that these corrections multiply  $\delta(1-z)$ , one immediately sees that the double poles  $1/\epsilon^2$ , of IR origin, cancel against the real diagrams, just as they should. However, note also that the single-pole structure is manifestly different; let us examine this a little more closely.

There is a substantial difference between a vector and a scalar vertex: the former is related to a conserved current, the latter not. Thus, the vector current is not renormalised while the scalar does receive radiative corrections. In other words, one must also take into account the renormalisation of the coupling constant (*i.e.*, the quark mass in the true Higgs case) associated with the vertices in consideration.<sup>3</sup> Indeed, the simplest way to evaluate the contribution is to calculate the quark-mass renormalisation, including the constant pieces. In the  $\overline{\text{MS}}$  scheme the standard calculation gives

$$\delta_m = -C_F \frac{\alpha_s(Q^2)}{2\pi} \left[ \frac{\mu^2}{-q^2} \right]^\epsilon \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \frac{3}{\epsilon} \frac{1}{1-2\epsilon}. \quad (12)$$

Including this as a contribution to the virtual corrections, one finally obtains

$$\delta_S^{\text{full}} = \delta_V. \quad (13)$$

Thus, combining real and virtual contributions, the complete set of coefficients for DIS are

$$C^{\text{DIS}}(z) = C_F \left\{ (1+z^2) \left[ \frac{\ln(1-z)}{1-z} \right]_+ - \frac{3}{2} \frac{1}{(1-z)_+} + 3 + 2z - (1+z^2) \frac{\ln z}{(1-z)} - \left[ \frac{9}{2} + \frac{\pi^2}{3} \right] \delta(1-z) \right\}, \quad (14a)$$

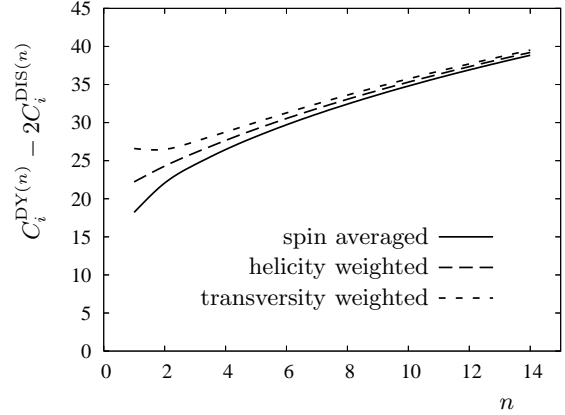
$$C_L^{\text{DIS}}(z) = C^{\text{DIS}}(z) - 1 - z, \quad (14b)$$

$$C_T^{\text{DIS}}(z) = C^{\text{DIS}}(z) - \frac{3}{2} - 2z - (1-z) \ln \left( \frac{1-z}{z} \right), \quad (14c)$$

## 2.4 The $K$ -Factor Results

The DY and DIS coefficients can now be combined to provide a theoretical  $K$  factor. Note that the DIS coefficient appears with a factor two in the required difference; this

<sup>3</sup> This observation was made by Blümlein [35] in regard of similar calculations aimed at evaluating the evolution kernel.



**Fig. 3.** The DY–DIS difference in the Mellin moments of the Wilson coefficients (*i.e.*, the  $K$  factor) for the three leading-twist densities.

merely reflects the two quarks (or rather quark–antiquark) in the initial state for DY. The final results are<sup>4</sup>

$$C^{\text{DY}}(z) - 2C^{\text{DIS}}(z) = C_F \left\{ 2(1+z^2) \left[ \frac{\ln(1-z)}{1-z} \right]_+ + \frac{3}{(1-z)_+} - 6 - 4z + \left[ \frac{4}{3}\pi^2 + 1 \right] \delta(1-z) \right\}, \quad (15a)$$

$$C_L^{\text{DY}}(z) - 2C_L^{\text{DIS}}(z) = C^{\text{DY}}(z) - 2C^{\text{DIS}}(z) + C_F 2(1+z), \quad (15b)$$

$$C_T^{\text{DY}}(z) - 2C_T^{\text{DIS}}(z) = C^{\text{DY}}(z) - 2C^{\text{DIS}}(z) + C_F \left\{ 7 - \frac{6z \ln^2 z}{(1-z)} - 2(1-z) \ln(1-z) \right\}, \quad (15c)$$

where the origins of the large differences in the last line may thus be traced in part to the different phase-space restrictions in the transversity case and in part to the residues due to the different splitting functions. It is perhaps worth reminding the reader that the bulk of the large  $K$  factor in the unpolarised and helicity cases (the  $\pi^2$  terms) comes from the necessary continuation of  $Q^2$  from space-like (in DIS) to time-like (in DY). However, the transversity correction contains other non-negligible pieces.

For a first visual comparison, Fig. 3 shows the Mellin moments, defined by

$$f^{(n)} \equiv \int_0^1 dx x^n f(x), \quad (16)$$

of the above differences in the Wilson coefficients between DY and DIS for the three leading-twist densities. While it is clear that the difference is generally only a little larger for transversity, the rapidly growing difference as  $n \rightarrow 0$  (equivalent in  $z$  space to  $z \rightarrow 0$ ) with respect to both the unpolarised *and* helicity cases is particularly striking.

<sup>4</sup> A brief review of the results presented here may be found in a recent contributed talk [36].

Note also that the small- $z$  (small- $n$ ) region counts heavily in the convolution integrals, therefore the corrections to the transversity-weighted cross-section can be significantly larger.

To provide an idea of the effect these corrections might have on an experimentally measured DY asymmetry, in Fig. 4 the helicity and transversity asymmetries for purely NS contributions (in both the numerator and denominator) are plotted as functions of  $\tau$ . The asymmetries are shown for a centre-of-mass energy  $\sqrt{s} = 200$  GeV, corresponding to recent polarised RHIC energies [4]. For the transversity distribution we have taken a conservative starting point of  $\Delta_T q(x, Q_0^2) = \Delta q(x, Q_0^2)$ . The evolution of the distributions has then been performed to LO here as the coefficient differences are scheme independent and the effect of higher orders on the  $K$  factor is negligible. The size of the shift due to the  $K$  factor is two to three times as large in the case of transversity with respect to helicity and typically reaches values of the order of 10% for these energies. It should be noted, however, that there is an automatic limitation of the  $K$ -factor difference (*e.g.*, with respect to large values of  $\alpha_s$  for low energies) owing to the presence of the  $\pi^2$  terms; when the  $K$ -factor difference between numerator and denominator becomes large in absolute terms so too do the overall  $K$  factors themselves, which to some extent cancels or dilutes the effect. Very similar corrections to those presented are obtained for energies corresponding to the proposed experiments at GSI. Repeating the calculations for various centre-of-mass energies, one finds that the corrections are maximal at  $\sqrt{s} \approx 40$  GeV, where for the transversity case they reach as much as 15%.

## 2.5 Cross-checks

A couple of simple cross-checks may be made on the influence of the scalar vertex itself. First of all, there is now the possibility of a purely Higgs, unpolarised DIS process (*i.e.*, in which both vertices are scalar). The contribution of the real diagrams is

$$\tilde{C}^{\text{DIS-R},S}(z) = \tilde{C}^{\text{DIS-R}}(z) - 2 - 3z, \quad (17)$$

which, combined with the virtual corrections already discussed, gives

$$C^{\text{DIS},S}(z) = C^{\text{DIS}}(z) - 2 - 3z. \quad (18)$$

The new coefficient in Eq. (18) could be used in place of the usual unpolarised correction to *define* the parton distributions.

Secondly, there is also similarly a possible purely scalar DY-like process; the NLO correction to the unpolarised cross-section in this case is found to be

$$C^{\text{DY},S}(z) = C^{\text{DY}}(z) + C_F 2(1 - z). \quad (19)$$

Moreover, Hikasa’s theorem is avoided here owing to the presence of the scalar vertices and a transverse-spin asymmetry is present even after integrating over the lepton-pair azimuthal angle. The NLO correction to the scalar

transversity asymmetry is

$$C_T^{\text{DY},S}(z) = C^{\text{DY}}(z) + C_F (1 - z)[4 - 4 \ln(1 - z) + 2 \ln z]. \quad (20)$$

In none of the above cases does the scalar vertex introduce large correction differences with respect to the vector. Indeed, in the last case of a purely scalar DY process (both spin averaged and transversely polarised), the only differences are residues of the difference in the LO splitting functions, as indicated by the form and the overall factor  $(1 - z)$ .

## 3 Conclusions

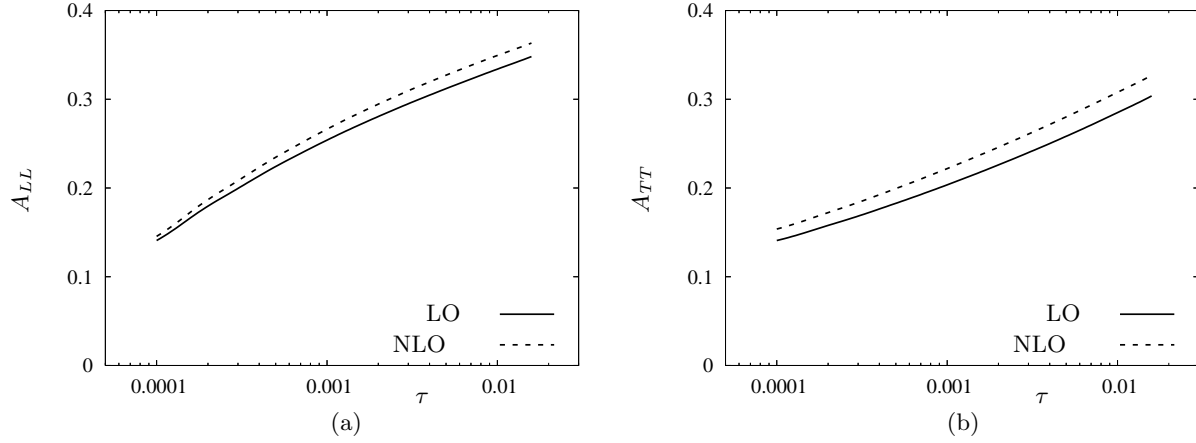
In order to appraise the real nature of the DY  $K$  factor in the case of transversity asymmetries, we have examined *gedanken* processes involving scalar vertices. This allows a natural DIS definition for the partonic densities  $\Delta_T q(x)$ . Such a definition allows a direct connection with model estimates based on knowledge of parton densities derived essentially from precisely DIS. Typical examples might be models in which at some low  $Q^2$  scale transversity- and helicity-weighted densities are naturally equal or others in which the Soffer bound [37] is found to be saturated, again at some low scale. In all such cases the spin-averaged and helicity-weighted densities used to set the starting point are obviously and naturally taken directly from DIS.

The results presented here provide a measure of the reliability of model predictions, without, of course, representing a rigorous estimate, in as much as the reference processes are partially fictitious and in any case are not precisely those normally adopted. However, we have seen that in general the corrections are not excessively large although they may be significantly larger than in the helicity case. Moreover, comparison of the Mellin moments indicates that in kinematical configurations in which low  $z = \tau/x_1 x_2$  dominates there could be very important corrections. On the other hand, many of the differences between NLO coefficients vanish numerically for  $z \rightarrow 1$  and so safe kinematical configurations certainly exist.

In closing then, although apparently fairly well under control, the question of NLO perturbative corrections in the case of transversely polarised DY processes clearly deserves more study, in particular, where the kinematics might be such experimentally as to favour the dangerous low- $z$  region.

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**Fig. 4.** The doubly polarised  $p\bar{p}$  helicity and transversity asymmetries  $A_{LL}$  and  $A_{TT}$  for purely valence-driven DY at LO and NLO as functions of  $\tau = Q^2/s$ , for  $\sqrt{s} = 200$  GeV.

## A Plus-regularised distributions

The so-called “plus-regularised” distributions are defined via integrals with a smooth test function  $f(y)$ :

$$\int_x^1 dy f(y) \left[ \frac{g(y)}{1-y} \right]_+ \equiv \int_x^1 dy \left[ \frac{f(y) - f(1)}{1-y} \right] g(y) - f(1) \int_0^x dy \frac{g(y)}{1-y}, \quad (21)$$

where  $g(y)$  is well-behaved as  $y \rightarrow 1$ .

## B Modified minimal subtraction scheme: implementation

The minimal subtraction (MS) scheme is defined, in conjunction with dimensional regularisation, as the removal of all simple poles in  $1/\epsilon$  (double and higher poles due to IR divergences are cancelled automatically between real and virtual contributions). However, common residual finite contributions are always left due to the appearance of the factor  $(4\pi)^\epsilon \Gamma(\epsilon)$ . Expanding to  $O(\epsilon)$ , one obtains

$$(4\pi)^\epsilon \Gamma(\epsilon) \simeq \Gamma(1+\epsilon) [1 + \epsilon \ln(4\pi)] \frac{1}{\epsilon} \simeq \frac{1}{\epsilon} + \ln(4\pi) - \gamma_E. \quad (22)$$

The  $\overline{\text{MS}}$  scheme then augments MS by subtracting the two  $\epsilon$ -independent terms above.

Thus,  $\overline{\text{MS}}$  may be implemented to  $O(\alpha_s)$  by defining the Feynman virtual momentum-integral measure  $[d^n k]$  to include a factor  $1/\Gamma(1+\epsilon)$  and by removing a factor  $(4\pi)^\epsilon$ . In other words, the plain MS definition may be substituted with the following:

$$[d^n k] \equiv \frac{1}{(4\pi)^\epsilon \Gamma(1+\epsilon)} \int \frac{d^n k}{(2\pi)^n}. \quad (23)$$

Consequently, the definition of the phase-space integral for a final two-body state (as in  $\gamma q \rightarrow qg$ ) must be modified analogously to

$$\text{PS}_2 \equiv \frac{1}{8\pi} s^{-\epsilon} \int_0^1 dy [y(1-y)]^{-\epsilon}, \quad (24)$$

where  $y = \frac{1}{2}(1 + \cos \theta)$  and  $\theta$  is the partonic centre-of-mass scattering angle (in this case between the incoming  $q$  and outgoing  $g$ ).

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